The paper is concerned with a very general version of Caristi’s coincidence theorem, in topological spaces \((X, \tau)\) satisfying some completeness conditions with respect to a function \(p : X \times X \to [0, \infty)\), and having as consequence Caristi’s fixed point theorem. The general result is applied to various concrete situations - quasi-metric spaces, bornological spaces - to obtain versions of Ekeland’s variational principle.

The paper contains some interesting results, but it is written in a negligent and superficial manner, so it needs a serious revision.

Below I do mention some oversights and misprints.

**Definition 2.1** Let \((X, \tau)\) be a topological space. An extended real-valued function \(f : X \to (-\infty, \infty]\) is said to be sequentially lower monotone if \(f(x) \leq f(x_n)\) for all \(n\), for every sequence \(\{x_n\}\) in \(X\) converging to \(x\) and satisfying

\[
f(x_1) \geq f(x_2) \geq \cdots \geq f(x_n) \geq \ldots
\]

Or

**Definition 2.1** Let \((X, \tau)\) be a topological space. An extended real-valued function \(f : X \to (-\infty, \infty]\) is said to be sequentially lower monotone if for every sequence \(\{x_n\}\) in \(X\) converging to \(x\) and satisfying

\[
f(x_1) \geq f(x_2) \geq \cdots \geq f(x_n) \geq \ldots,
\]

we have \(f(x) \leq f(x_n)\) for all \(n\).

**in Definition 2.2**: please say: "Let \((X, \tau)\) be a topological space and \(p : X \times X \to [0, \infty)\) a function."

at the end appears \(f(\bar{x})\) instead of \(f(x)\).

**in Definition 2.3**

"be a a function” there is and extra ”a” (in Def.2.2 as well)

(1) \((X, \tau)\) is said to be \(p\)-complete ([11],[12]) if any sequence
in (2) "... w.r.t. p if \((X, \tau)\) is \(p\)-complete and
\[
\liminf_{n \to \infty} p(x_n, y) \leq p(x, y) \quad (2.1)
\]
for any sequence \(\{x_n\}\) in \(X\) is convergent to some \(x\) and satisfying \(\sum_{n=1}^{\infty} p(x_n, x_{n+1}) < \infty\).

(3) Let \(f: X \to (-\infty, \infty]\) be a proper function (i.e., \(f(x)\) is not identically \(+\infty\)). The topological space \((X, \tau)\) is said to be sequentially lower complete w.r.t. \(p\) and \(f\) if any sequence \(\{x_n\}\) in \(X\) satisfying \(\sum_{n=1}^{\infty} p(x_n, x_{n+1}) < \infty\) and \(f(x_n) \leq f(x_{n+1})\) for all \(n\), is convergent to some \(x \in X\) and (2.1) holds for all \(y \in X\).

NOTE. If the notion of proper function is explained here, there is no need to repeat it in all the theorems, that is cut this explanation from Th. 2.1, 2.2, etc.

Concerning REMARK 2.1
A function \(f\) is:
\[
\text{lsc at } x \in X \text{ iff } f(x) \leq \liminf_{y \to x} f(y)
\]
\[
\text{usc at } x \in X \text{ iff } f(x) \geq \limsup_{y \to x} f(y)
\]
Consequently the assertion concerning the sequential upper semi-continuity of \(p(\cdot, y)\) does not fit with (2.1)
Please take care of this matter.

page 3
in Th. 2.1
"a function" not "functions"
delete "if" at the end of line 5 in this Thm
it is better to denote by \(i_0\) the element of \(I\) (instead of \(l_0\), in all places)
also \(y \in T_{i_0}(x) \setminus \{g(x)\}\) not \(y \in T_{i_0}(x) / \{g(x)\}\) (in all places)
the last line: replace "such that" by "and" and cut the comma

in the Proof of Th.2.1
"We take \(x_0 \in X\). Since \(f\) is proper we can assume, without loss of generality, that \(f(x_0) < \infty\)." (there is no need for further explanation).
Concerning the inequality
\[
\frac{1}{2}(f(x_0) + \inf_{x \in S_1} f(x)) < f(x_0).
\]
In spite of the fact that \(x_0 \notin S_1\), it possible that \(\inf_{x \in S_1} f(x) = f(x_0)\), in which case the above inequality becomes equality. The same for (2.4). The strict inequality is necessary to obtain the contradiction in the proof.
This must be fixed.
page 5
first row: Combining with (2.6) in Th. 2.2;
Delete ”if” in ”Suppose further that if for...”
Corol. 2.1
”Suppose that \( \eta : [0, \infty) \rightarrow (-\infty, \infty) \) satisfies \( \eta(0) = 0 \) and that \( \varphi....”
put a comma at the end of the formula (2.8) and continue next row with ”and one of the following conditions is satisfied:”

page 6
” Then each Caristi type map...” (delete ”for”)
line 6 ”Then, as in the proof of Theorem 1 in [16],...”
”complete” instead of ”completed” (the same in line 14 from the bottom)
Start ”Case (ii). ” as a paragraph

page 7
in Th.2.3 ”a function” not ”functions”
in (II) delete ”if” ”Suppose further that for each...”

page 10
last row: ”Cauchy” not ”Cauchy”

page 12
line 6 from the top: I think that the reference is to Remark 3.1, not 3.2

in the REFERENCES

General remark: some journals are written with full names, while others in abbreviated form. This must be done according to journal’s style.

item [10] Equivalents instead of Equivalent
item [20] Ekeland’s
item [22] Monatshefte für Mathematik