

Methodology article

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Full cyclic coordinate descent: solving the protein loop closure problem in $C\alpha$ space

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Abstract

Background: Various forms of the so-called *loop closure problem* are crucial to protein structure prediction methods. Given an N- and a C-terminal end, the problem consists of finding a suitable segment of a certain length that bridges the ends seamlessly.

In homology modelling, the problem arises in predicting loop regions. In *de novo* protein structure prediction, the problem is encountered when implementing local moves for Markov Chain Monte Carlo simulations.

Most loop closure algorithms keep the bond angles fixed or semi-fixed, and only vary the dihedral angles. This is appropriate for a full-atom protein backbone, since the bond angles can be considered as fixed, while the (ϕ , ψ) dihedral angles are variable. However, many *de novo* structure prediction methods use protein models that only consist of $C\alpha$ atoms, or otherwise do not make use of all backbone atoms. These methods require a method that alters both bond and dihedral angles, since the pseudo bond angle between three consecutive $C\alpha$ atoms also varies considerably.

Results: Here we present a method that solves the loop closure problem for $C\alpha$ only protein models. We developed a variant of Cyclic Coordinate Descent (CCD), an inverse kinematics method from the field of robotics, which was recently applied to the loop closure problem. Since the method alters both bond and dihedral angles, which is equivalent to applying a full rotation matrix, we call our method Full CCD (FCDD). FCCD replaces CCD's vector-based optimization of a rotation around an axis with a singular value decomposition-based optimization of a general rotation matrix. The method is easy to implement and numerically stable.

Conclusion: We tested the method's performance on sets of random protein $C\alpha$ segments between 5 and 30 amino acids long, and a number of loops of length 4, 8 and 12. FCCD is fast, has a high success rate and readily generates conformations close to those of real loops. The presence of constraints on the angles only has a small effect on the performance. A reference implementation of FCCD in Python is available as supplementary information.

Background

Many protein structure prediction methods require an

algorithm that is capable of constructing a new conformation for a short segment of the protein, without affecting

the rest of the molecule. In other words, a protein fragment needs to be generated that seamlessly closes the gap between two given, fixed end points. This problem is generally called the *loop closure problem*, and was introduced in a classic paper by Go and Scheraga more than 30 years ago [1]. It has been the continued subject of intensive research over many years due to its high practical importance in structure prediction.

The loop closure problem arises in at least two different structure prediction contexts. In homology modelling, it is often necessary to rebuild certain loops that differ between the protein being modelled and the template protein [2]. The modelled loop needs to bridge the gap between the end points of the template's loop.

In *de novo* prediction, *local resampling* or *local moves* can be considered as a variant of the loop closure problem. Typically, the conformation of a protein segment needs to be changed without affecting the rest of the protein as a sampling step in a Markov Chain Monte Carlo (MCMC) procedure [3]. In both homology and *de novo* structure prediction, the problem is however essentially the same.

The classic article by Go and Scheraga [1] describes an analytical solution to finding all possible solutions for a protein backbone of three residues. In this case, the degrees of freedom (DOF) comprise six dihedral angles, i.e. the backbone's (ϕ , ψ) angles. Another approach is to use a fragment library derived from the set of solved protein structures, and look for fragments or combinations of fragments that bridge the given fixed ends [4-6]. More recently, the loop closure problem has been tackled using algorithms borrowed from the field of robotics, in particular inverse kinematics methods [7-9]. Still other methods use various Monte Carlo chain perturbation approaches, often combined with analytical methods [10,11,3,12]. A good overview of loop closure methods and references can be found in Kolodny *et al.* (2005) [6].

Most methods assume that one is working with a full-atom protein backbone with fixed bond angles and bond lengths, so the DOF consist solely of the backbone's (ϕ , ψ) angles. However, in many cases not all the atoms of the protein backbone are present in the model. In particular, a large class of structure prediction, design and *in silico* folding methods makes use of drastically simplified models of protein structure [13,14].

A protein structure might for example be represented by a chain of $C\alpha$ atoms or a chain of virtual atoms at the centers of mass of the side chain atoms [15]. In these models, there is obviously no full-atom model of the protein's backbone available.

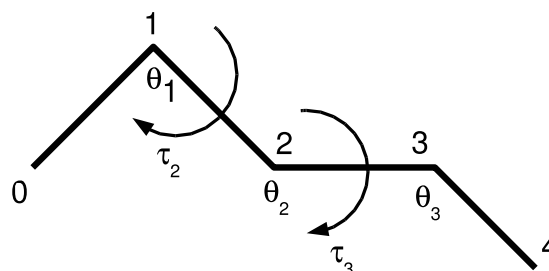


Figure 1

A protein segment's $C\alpha$ trace. The $C\alpha$ positions are numbered, and the pseudo bond angles θ and pseudo dihedrals τ are indicated. The segment has length 5, and is thus fully described by two pseudo dihedral and three pseudo bond angles.

In the case of $C\alpha$ -only models, the structure can be described as a sequence of pseudo bonds, pseudo angles θ and pseudo dihedral angles τ [16]. Here, the term 'pseudo' indicates that the consecutive $C\alpha$'s are not actually connected by chemical bonds. As in the case of the protein's backbone, the pseudo bond lengths can be considered fixed (typically 3.8 Å). In contrast, the pseudo bond angles between three consecutive $C\alpha$ atoms are most definitely not fixed, but vary between 1.4 and 2.7 radians. Hence, a $C\alpha$ -only model of N residues can be represented by a sequence of $N - 2$ pseudo bond angles θ and $N - 3$ pseudo dihedral angles τ (Figure 1).

Most inverse kinematics approaches assume that the DOF consist only of dihedral angles, and keep the bond angles fixed or semi-fixed. Hence, they cannot be readily applied to the $C\alpha$ -only case without restricting the search space unnecessarily. In principle, fragment library based methods would apply, but here the problem of data sparsity arises [17,18]. Often, no suitable fragments can be found if the number of residues between the fixed ends becomes too high.

In order to solve the loop closure problem in $C\alpha$ space, we extend a particularly attractive approach that was recently introduced by Canutescu & Dunbrack [8]. The algorithm is called Cyclic Coordinate Descent (CCD), and like many other loop closure algorithms it derives from the field of robotics [19]. As pointed out by Canutescu & Dunbrack, the CCD algorithm is meant as a black box method that generates plausible protein segments that bridge two given, fixed endpoints. The final choice is typically made based upon the occurrence of steric clashes, applicable

constraints (for example side chain conformations) and evaluation of the energy.

The CCD algorithm does not directly generate conformations that bridge a given gap, but alters the dihedral angles of a given starting segment that already overlaps at the N-terminus such that it also closes at the C-terminus. The starting segment can be generated in many ways, for example by using a fragment library derived from real structures or by constructing random artificial fragments with reasonable conformations. Surprisingly, most protein loops can be closed efficiently by CCD starting from artificial loops constructed with random (ϕ, ψ) dihedral angles [8].

The CCD algorithm alters the (ϕ, ψ) dihedral angles for every residue in the segment in an iterative way. In each step, the RMSD between the chain end and the overlap is minimized by optimizing one dihedral angle. Because only one dihedral angle is optimized at a time, the optimal rotation can be calculated efficiently using simple vector arithmetic.

The list of advantages of CCD is impressive: it is conceptually simple and easy to implement, computationally fast, very flexible (ie. capable of incorporating various restraints and/or constraints) and numerically stable. Therefore, we decided to adopt the CCD algorithm for use with $C\alpha$ -only models. Here, we describe a new version of CCD that optimizes both dihedral angles and bond angles, while maintaining all the advantages of the CCD method. We call our method Full Cyclic Coordinate Descent (FCCD), where "Full" indicates that both dihedral angles and bond angles are optimized, while only the bond lengths remain fixed. At the heart of the FCCD method lies a procedure to superimpose point sets with minimal Root Mean Square Deviation (RMSD), based on singular value decomposition. As is the case for the CCD algorithm, FCCD is not a modelling method in itself. Rather, it can be used as a method to generate possible conformations that can be evaluated using some kind of energy function.

To test the algorithm, we selected random segments from a protein structure database, and evaluated the efficiency of closing the corresponding gaps starting from artificial segments with protein-like (θ, τ) angles. We show that FCCD is both fast and successful in solving the loop closure problem, even in the presence of angle constraints. Conformations close to those of real protein loops are readily generated. Finally, we discuss possible applications of the FCCD algorithm, and mention some possible disadvantages.

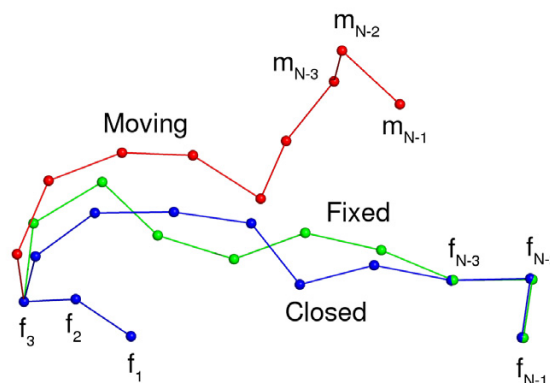


Figure 2

The action of the FCCD algorithm in $C\alpha$ space. The $C\alpha$ traces of the moving, fixed and closed segments are shown in red, green and blue, respectively. The $C\alpha$ atoms are represented as spheres. The labels f_0, f_1 and f_2 indicate the three fixed vectors at the N-terminus that are initially common between the fixed and moving segments. The loop is closed when the three C-terminal vectors of the moving segment (labelled $m_{N-3}, m_{N-2}, m_{N-1}$) superimpose with an RMSD below the given threshold on the three C-terminal vectors of the fixed segment (labelled $(f_{N-3}, f_{N-2}, f_{N-1})$). This figure and Figure 3 were made with PyMol <http://www.pymol.org>.

Results and discussion

Overview of the FCCD algorithm

Figure 2 illustrates the essence of the FCCD algorithm, and Table 3 provides detailed pseudo code. Here we define some of the terms that will be used throughout the article, and provide a high level overview of the FCCD algorithm.

The *fixed segment* is a list of $C\alpha$ vector positions that specifies the gap that needs to be bridged. Only the first and last three $C\alpha$ positions, with corresponding vectors (f_0, f_1, f_2) and $(f_{N-3}, f_{N-2}, f_{N-1})$ are relevant. We will call these two sets of vectors the *N- and C-terminal overlaps*, respectively. The *moving segment* is a list of $C\alpha$ position vectors that will be manipulated by the FCCD algorithm to bridge the gap. The *closed segment* is the moving segment after its pseudo bond angles and pseudo dihedral angles were adjusted to bridge the N- and C-terminal overlaps of the fixed segment. The vectors describing the positions of the $C\alpha$ atoms in a segment of N residues are labelled from 0 to $N - 1$.

Initially, the first three vectors of the moving loop coincide with the first three vectors of the fixed segment, while the last three vectors are conceivably reasonably close to the last three vectors of the fixed loop. This last condition is however not very critical. The moving segment can be obtained using any algorithm that generates plausible $C\alpha$ fragments, including deriving them from real protein structures. The fixed segment is typically derived from a real protein of interest, or a model in an MCMC simulation.

The FCCD algorithm changes the pseudo bond angles and pseudo dihedral angles of the moving loop in such a way that the RMSD between the last three vectors of the moving loop ($m_{N-3}, m_{N-2}, m_{N-1}$) and the last three vectors of the fixed loop ($f_{N-3}, f_{N-2}, f_{N-1}$) is minimized, thereby seamlessly closing the gap.

Note that we assume that the last three vectors of the moving and fixed segments can be superimposed with an RMSD of 0.0 Å (see Figure 2). In other words, the first and last pseudo bond angles in both segments are equal. It is however perfectly possible to use segments with different pseudo bond angles at these positions. Since the final possible minimum RMSD will be obviously greater than 0 in this case, the RMSD threshold needs to be adjusted accordingly.

The algorithm proceeds in an iterative way. In each iteration, a vector m_i in the moving segment is chosen that will serve as a center of rotation. This chosen center of rotation will be called the pivot throughout this article. Then, the rotation matrix that rotates ($m_{N-3}, m_{N-2}, m_{N-1}$) on ($f_{N-3}, f_{N-2}, f_{N-1}$) around the pivot and resulting in minimum RMSD is determined, and applied to all the vectors m_j downstream i (with $i < j < N$). In the next iteration, a new pivot is chosen, and the procedure is repeated. The vectors in the chain can be traversed linearly, or they can be chosen at random in each iteration. The difference between FCCD and CCD is that the latter applies a general rotation to the chain using an atom in the chain as a pivot, while the former only applies a rotation around a single axis. The process is stopped when the RMSD falls below a given threshold.

Finding the optimal (with respect to the RMSD) rotation matrix corresponds to finding one optimal pseudo bond angle and pseudo dihedral angle pair. We define θ_i as the bond angle of the vectors m_{i-1}, m_i, m_{i+1} and τ_i as the dihedral angle of the vectors $m_{i-2}, m_{i-1}, m_i, m_{i+1}$ (see Figure 1 and [16]). These definitions have the intuitive interpretation that altering (θ_i, τ_i) changes the positions of all $C\alpha$'s downstream from position i . Conversely, using pivot m_i and applying a rotation matrix to all the positions down-

stream from position i corresponds to changing pseudo bond angle θ_i and pseudo dihedral angle τ_i .

For a segment of N $C\alpha$'s (with $N > 3$), the pseudo angles range from θ_1 to θ_{N-2} and the pseudo dihedrals range from τ_2 to τ_{N-2} . Since the first and last bond angles of the moving segment are fixed, the pivot points range from position 2 to position $N - 3$ (with $N > 4$). The pseudo bond angle and pseudo dihedral angle pairs thus range from (θ_2, τ_2) to (θ_{N-3}, τ_{N-3}).

Finding the optimal rotation matrix with respect to the RMSD of the C-terminal overlaps can be efficiently solved using singular value decomposition, as described in detail in the following section.

Finding the optimal rotation

In this section we discuss solving the following subproblem arising in the FCCD algorithm: given a chosen pivot point i in the moving segment, find the optimal (θ_i, τ_i) pair that minimizes the RMSD between the last three $C\alpha$ vectors in the moving segment and the last three $C\alpha$ vectors in the fixed segment. Recall that the (θ_i, τ_i) pair at position i corresponds to the pseudo bond angles and pseudo dihedral angles defined by vectors m_{i-1}, m_i, m_{i+1} and $m_{i-2}, m_{i-1}, m_i, m_{i+1}$ respectively.

Finding the optimal (θ_i, τ_i) pair simply corresponds to finding the optimal rotation matrix using $C\alpha$ position i as the center of rotation (see Figure 2). This reformulated problem can be solved by a variant of a well known algorithm to superimpose two point sets with minimum RMSD which makes use of singular value decomposition [20,21]. Below, we describe this adapted version of the algorithm.

First, the C-terminal overlaps of the moving and the fixed segment need to be translated to the new origin that will be used as pivot for the optimal rotation. This new origin is the pivot vector m_i at $C\alpha$ position i in the moving segment. The new vector coordinates of the moving and the fixed segments are put in two matrices (respectively M and F), with the coordinates of the vectors positioned column wise:

$$M = [m_{N-3} - m_i \mid m_{N-2} - m_i \mid m_{N-1} - m_i]$$

$$F = [f_{N-3} - m_i \mid f_{N-2} - m_i \mid f_{N-1} - m_i]$$

Then, the correlation matrix Σ is calculated using M and F :

$$\Sigma = FM^T$$

Any real $n \times m$ matrix A can be written as the product of an orthogonal $n \times n$ matrix U , a diagonal $n \times m$ matrix D and an orthogonal $m \times m$ matrix V^T [22]. Such a factorization is called a *singular value decomposition* of A . The positive diagonal elements of D are called the *singular values*. Hence, Σ can be written as:

$$\Sigma = UDV^T$$

The optimal rotation Γ is then calculated as follows:

$$\Gamma = USV^T$$

The value of the diagonal 3×3 matrix S is determined by the product $\det(U)\det(V^T)$, which is either 1 or -1. If this product is -1 then $S = \text{diag}(1, 1, -1)$, else S is the 3×3 unit matrix. The matrix S ensures that Γ is always a pure rotation, and not a rotation-inversion [21].

In order to apply to all the vectors that are downstream from the pivot point i , these vectors are first translated to the origin of the rotation (ie. pivot point m_i), left multiplied by Γ and finally translated back to the original origin:

$$m_j^\Gamma = \Gamma(m_j - m_i) + m_i$$

where $i < j < N$.

Adding angle constraints to FCCD

It is straightforward to constrain the (θ, τ) angles to a given probability distribution. For each rotation matrix Γ , the resulting new pseudo bond angles and dihedral angles can easily be calculated. The new angles can for example be accepted or rejected using a simple rejection sampling Monte Carlo scheme, comparing the probabilities of the previous pair $(\theta^{prev}, \tau^{prev})$ with that of the next pair $(\theta^{next}, \tau^{next})$. If $P(\theta^{next}, \tau^{next}) > P(\theta^{prev}, \tau^{prev})$ the change is accepted, otherwise it is accepted with a chance proportional to $P(\theta^{next}, \tau^{next}) / P(\theta^{prev}, \tau^{prev})$. A similar approach was used by Canutescu & Dunbrack [8], and we describe

its performance in combination with FCCD in the following section.

More advanced methods could take the probability of the sequence of angles into account as well, for example using a Hidden Markov Model of the backbone [23]. The pseudo code in Table 3 illustrates accepting/rejecting rotations using an unspecified 'accept' function, whose details will depend on the application.

FCCD's performance

In order to evaluate the general efficiency of the method, we selected random fragments of various sizes from a representative database of protein structures, and used these fragments as fixed segments. Hence, the evaluation described below is not limited to loops, but extends to random protein segments. This is a relevant test, since local moves in a typical MCMC simulation are indeed performed on random segments.

The fixed segments were sampled from a dataset of fold representatives (see Methods). First we selected a random fold representative, and subsequently extracted a random continuous fragment of suitable length. The lengths varied from 10 to 30 with a step size of 5. It should be noted that the length of the segment here refers to the number of $C\alpha$ atoms between the ends that need to be bridged.

The moving segments were generated using random dihedral and bond angles in regions accessible to proteins (see previous section). This was done by sampling the (θ_i, τ_i) pairs according to a probability distribution derived from a set of representative protein structures (see Methods). The bond length was fixed at 3.8 Å, in tune with the consensus $C\alpha-C\alpha$ distance in protein structures. The last bond angle in the moving segment was chosen equal to the last bond angle in the fixed loop to make a final RMSD of 0.0 Å possible. The RMSD threshold was 0.1 Å. The maximum number of iterations was set to 1000, where one iteration is a sweep over all positions. We ran the FCCD program on 1000 different fixed segments. Table 1 summarizes the results.

Table 1: Performance of the FCCD algorithm for various segment lengths. The first and second number in columns 2–4 refer to unconstrained and constrained FCCD, respectively. Columns 2 and 3 respectively show the average time and number of iterations needed for closing a single segment successfully. The percentage of loops successfully closed in under 1000 iterations is shown in the last column.

Segment length	Average time (ms)	Average iterations	% Closed
5	4.5/51.7	14.0/27.0	99.90/86.50
10	5.2/28.3	10.5/16.8	99.40/98.20
15	5.6/28.6	7.8/12.1	99.60/99.40
20	6.2/27.1	6.3/9.0	99.80/99.40
25	7.6/31.7	5.5/7.6	99.00/99.90
30	7.1/31.0	4.4/6.3	99.70/99.40

A first observation is the effect of the angle constraints. These slow down FCCD with a factor of 10 for small segments (5 residues) and roughly a factor of 5 for larger segments (10 residues or more). Nonetheless FCCD including constraints remains quite speed efficient: small five residue segments are on average closed in about 50 ms, while larger segments (from 10 to 30 residues) are closed considerably faster (on average in about 30 ms). The explanation for this is of course that it is easier to close large segments because they have more DOF. Hence, FCCD, like CCD, is fast and easily handles large segments efficiently.

Overall, the success rate of FCCD is excellent, and very little affected by constraints. For 5 residue segments, adding constraints diminishes the number of successfully closed segments from 99.9% to 86.5%. This effect is however much less pronounced for larger segments: more than 98% percent of the moving/fixed segment pairs can be successfully closed. In short, FCCD is both speed efficient and has a high success rate, even in the presence of constraints.

Evaluation of FCCD's sampling space

Does FCCD potentially generate realistic protein conformations? FCCD could be used to propose possible conformations that are subsequently evaluated by an energy function. In this context, it is of course imperative to generate realistic conformations. To answer this question, we evaluate FCCD's ability to generate closed segments that are close to real protein loops. We used 30 real loops with lengths of 4, 8 and 12 residues as fixed segments. The loop length refers to the number of residues between the N- and C-terminal overlaps.

FCCD was applied using (θ, τ) constraints and an RMSD threshold of 0.1 Å. The maximum number of iterations was set to 1000. For each loop, we attempted to generate closed segments from 1000 random moving segments within the allowed number of iterations. The moving segments were generated as described in the previous section. For all 30 loop cases, we then identified the closed segment that resembled the input loop best as judged by the RMSD. For the calculation of the RMSD, we included the N- and C-terminal overlaps. The results are shown in Table 2, and the best fitting loops for each loop size are shown in Figure 3.

It is clear that FCCD readily generates closed segments that are reasonably close to the real loops, with an average RMSD of about 0.6, 2.2 and 3.0 Å for loops of 4, 8 and 12 residues, respectively. The highest minimum RMSD values for these loop lengths are 0.76, 2.42 and 3.37 Å, respectively, indicating that FCCD in general can come up with a reasonably close conformation. Using more initial

moving segments will obviously increase the chance of encountering a close conformation. Additionally, one can also expect an even better performance with a more refined way to constrain the (θ, τ) angles.

Conclusion

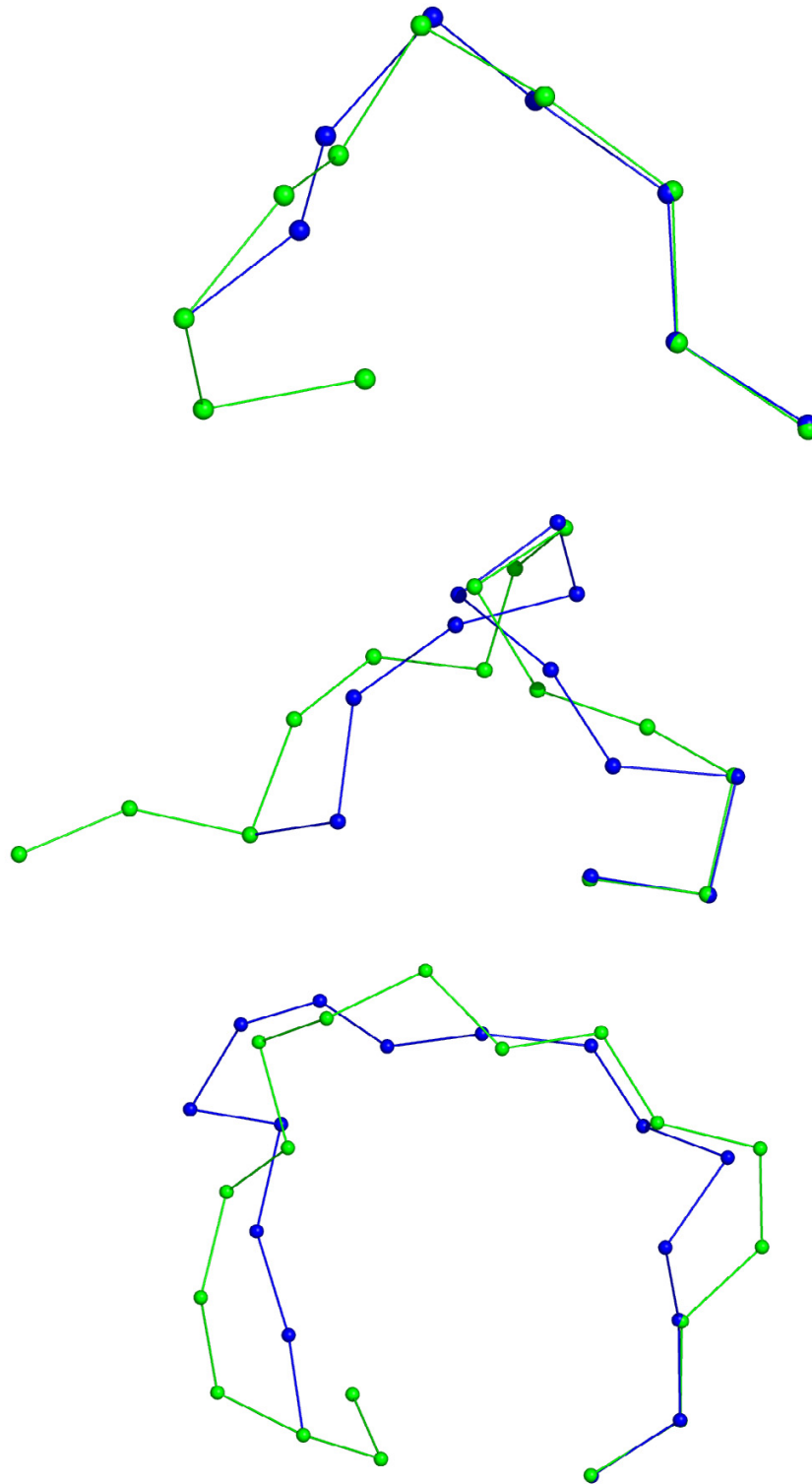
In this article, we introduce an algorithm that solves the loop closure problem for $C\alpha$ only protein models. The method is conceptually similar to the CCD loop closure method introduced by Canutescu and Dunbrack [8], but optimizes dihedral and bond angles simultaneously, while the former method only optimizes one angle at a time. At the heart of the method lies a modified algorithm to superimpose point sets with minimum RMSD, based on singular value decomposition [20,21].

The algorithm is fast, numerically stable and leads to a solution for the great majority of loop closure problems studied here. Importantly, the method remains efficient even in the presence of constraints on the dihedral and bond angles. FCCD readily handles large gaps, and potentially generates realistic conformations. Compared to other loop closure methods, FCCD is surprisingly easy to implement provided a function is available to calculate the singular value decomposition of a matrix.

A possible disadvantage is that FCCD has a tendency to induce large changes to the pseudo angles at the start of the moving segment while angles near the end are less affected, which is also the case for CCD [8]. This can for example be avoided by selecting the pivot points in a random fashion, or by limiting the allowed change in the angles per iteration. Occasionally the method gets stuck, which can be avoided by incorporating stochastic changes away from the encountered local minimum. One can also simply try again with a new random moving segment. We believe that CCD and FCCD despite these disadvantages are among the most efficient loop closure algorithms currently available.

The FCCD algorithm proposed here has great potential for use in structure prediction methods that only make use of $C\alpha$ atoms, or that otherwise do not include all backbone atoms [15,13,14]. FCCD could be used for example to implement local moves in a MCMC procedure. The moving segments could be derived from a fragment database or generated from a probabilistic model of the protein backbone. The latter model could range from a primitive probability distribution over allowed (θ, τ) angle pairs like we used here to a Hidden Markov Model that also models the sequence of (θ, τ) angle pairs.

We are planning to use the FCCD algorithm in combination with a sophisticated probabilistic model of the protein's backbone, which will steer both the generation of

**Figure 3**

Loops generated by FCCD (blue) that are close to real protein loops (green). The loops with lowest RMSD to a given loop of length 4 (top), 8 and 12 (bottom) are shown (loops lqnr, A, 195–198, 3chb, D, 51–58 and lctq, A, 26–37). The N- terminus is at the left hand side.

Table 2: Minimum RMSD (out of 1000 tries) between a fixed segment derived from a protein structure and a closed segment generated by FCCD. The length of the loops is shown between parentheses in the upper row.

Loop (4)	RMSD	Loop (8)	RMSD	Loop (12)	RMSD
ldvj, A, 20–23	0.59	lcru, A, 85–92	2.31	lcru, A, 358–369	3.37
ldys, A, 47–50	0.67	lctq, A, 144–151	2.22	lctq, A, 26–37	2.40
legu, A, 404–407	0.61	ld8w, A, 334–341	2.04	ld4o, A, 88–99	3.20
lej0, A, 74–77	0.61	lds1, A, 20–27	2.20	ld8w, A, 43–54	2.74
li0h, A, 123–126	0.73	lgk8, A, 122–129	2.20	lds1, A, 282–293	3.16
lid0, A, 405–408	0.66	li0h, A, 145–152	2.42	ldys, A, 291–302	2.90
lqnr, A, 195–198	0.54	lixh, 106–113	1.98	legu, A, 508–519	3.06
lqop, A, 44–47	0.58	llam, 420–427	2.16	lf74, A, 11–22	3.12
ltca, 95–98	0.76	lqop, B, 14–21	2.17	lqlw, A, 31–42	3.04
lthf, D, 121–124	0.56	3chb, D, 51–58	1.97	lqop, A, 175–186	2.97
Average RMSD	0.63	Average RMSD	2.17	Average RMSD	3.00

Table 3

```

maxit = maximum number of iterations
moving = N × 3 matrix of Cα positions in moving segment
fixed = N × 3 matrix of Cα positions in fixed segment
threshold = desired minimum RMSD
N = length of the segments
M = 3 × 3 matrix (centered coordinates along columns)
F = 3 × 3 matrix (centered coordinates along columns)
S = diag(1, 1, -1)
repeat maxit:
  # Start iteration over pivots
  for i from 2 to N-3:
    pivot = moving[i,:]
    # Make pivot point origin
    for j from 0 to 2:
      M[:,j] = moving [N-3+j,:]-pivot
      F[:,j] = fixed [N-3+j,:]-pivot
    # Find the rotation Γ that minimizes RMSD
    Σ = FMT
    U, D, VT = svd(Σ)
    # Check for reflection
    if det(U)det(VT)<0:
      U = US
    Γ = UVT
    # Evaluate and apply rotation
    if accept(Γ):
      # Apply the rotation to the moving segment
      for j from i+1 to N-1:
        moving [j,:] = Γ (moving [j,:]-pivot)+pivot
      rmsd = calc_rmsd(moving [N-3:], fixed [N-3:])
      # Stop if RMSD below threshold
      if rmsd<threshold:
        return moving, rmsd
# Failed: RMSD threshold not reached before maxit
return 0

```

The accept function rejects or accepts the proposed rotation, based on the resulting (θ , τ) pair. The svd function performs singular value decomposition, and calc_rmsd calculates the RMSD between two lists of vectors.

Table 4: SABMark identifiers of the 236 structures used as fold representatives

lew6a_	lail_	l11la_	lkid_	ln8yc1	lgzhl	le5da1	lep3b2	lihoa_	lm0wa1
ldhs_	lgpua2	2lefa_	lnsta_	leaf_	liiba_	ld5ra2	lfoha3	lgpua3	lcrza2
3pvia_	li6pa_	le4ft1	lkx5d_	2pth_	llu9a2	ldkla_	lfsqa_	lm2oa3	2dpma_
lajsa_	lfxoa_	3tg1_	lhx4a_	lmtyg_	lduvg2	lqopb_	liata_	lk2yx2	lf0ka_
layl_1	ltoaa_	8abp_	lnh8a1	lbi5a2	2mhr_	la2pa_	3lzt_	ldkia_	le7la2
lb4fa_	lbb8_	lkpf_	lmu5a2	lfda_	lgpea2	ljqa_	la2va2	ljfma_	l117a2
lcjxa1	llo7a_	lfm0e_	lfs1b2	lo0wa2	ldtja_	lk0ra3	levsa_	ljpdx2	lqdl1a1
ld5ya3	lh3fa2	liq0a3	ltig_	lxxaa_	lck9a_	lgyxa_	le5qa2	livsa2	lqbea_
3grs_3	lf08a_	lc7ka_	llkka_	ldq3a3	luox_1	l2asa_	lbob_	lm4ja_	ldv5a_
lf5ma_	lk2ea_	lei1a2	ljdw_	l1n1a_	2pola2	lf0ia1	lr16a1	lfvia2	lj7la_
lis2a1	le8ga2	lqr0a1	2dnja_	lkuua_	lqh5a_	lii7a_	lb8pa2	lj7na3	lchua3
lf00i3	lgri_1	lnkd_	lmwxa3	lj4a_	lih7a2	leula2	lgnla_	lmaz_	2por_
4htci_	les7b_	ltocr1	ld11a_	lfd3a_	li8na_	lh8pa1	4sgbi_	lftv_	lquba1
ld4va3	ltpg_2	liuaa_	lfv5a_	lmdya_	lzmecl	lfign_	leska_	li50i2	lfbva4
ldmc_	le53a_	lezvb1	ljeqa1	lk3ea_	lrec_	llm5a_	lk82a1	ljaja_	lm0ka_
lc0va_	lkqfc_	locrk_	lh67a_	2cpga_	ljra1	lbrwa1	lhs7a_	2cbla2	ljmxa2
lhyp_	lcuk_2	lecwa_	li9la_	lg7da_	ljkw_1	ldgna_	liqpa1	lpa2a_	lko9a1
lflza1	lks9a1	2sqca2	ld2ta_	lh3la_	lwer_	lb3ua_	ln1ba2	lpoc_	le79i_
lm1qa_	lenwa_	lg4ma1	le5ba_	lqhoa2	lkv7a2	li4ia2	lc8da_	lamm_1	lca1_2
lphm_2	ld7pm_	ljicb2	lfcla1	lgr3a_	lmjsa_	la8d_1	lff6a2	lfqta_	ljb0e_
ljh2a_	llyal_	lmgqa_	lhcia1	lb3qa2	lj1xa1	ldar_1	lexma2	lejea_	lagja_
le79d2	2rspa_	lh0ha1	lgral_	2erl_	lbtn_	lff7a_	ljmxa5	lcrua_	lm1xa4
lhx0a1	lgoia1	lcicy_2	ldaba_	3tdt_	lgg3a1	lpmi_	lbd0_	lh3ia2	lgppa_
lf39a_	lk6wa1	ljqna_	llu9a1	lm6ia1	lo94a3				

the initial moving loop and the acceptance/rejection of the angles. The performance of FCCD in this context will be the subject of a future publication.

Methods

Implementation

The FCCD algorithm was implemented in C, using the LAPACK [24] function dgesvd for the calculation of the singular value decomposition. Handling PDB files and calculating the (θ, τ) angles [16] was done using Biopython's Bio.PDB module [25]. We used a 2.5 GHz Pentium processor to calculate the benchmarks. A reference implementation of FCCD in Python is available as supplementary information.

Structure databases

For the calculation of the (θ, τ) probability distribution and the generation of random protein fragments, we used the SABMark 1.63 Twilight Zone database [26]. SABMark Twilight Zone contains 2230 high quality protein structures, divided over 236 different folds. All protein pairs have a BLAST E-value below 1, and thus presumably belong to different superfamilies. A dataset of fold representatives was generated by selecting a single structure at random for each fold (see Table 4).

The loops used to evaluate FCCD's sampling space were derived from Canutescu & Dunbrack [8]. We shifted two loops (1d8w, A, 46–57 and 1qop, A, 178–189) by three

residues to ensure that all loops had three flanking residues on each side.

Calculation of the (θ, τ) probability distribution

The bond angle θ was subdivided in 18 bins and the dihedral angle τ in 36 bins, in both cases starting at 0 degrees and with a bin width of 10 degrees. All (θ, τ) angles were extracted from all structures in the SABMark Twilight Zone database that consisted of a polypeptide chain without breaks. In total, 257534 angle pairs were extracted. Each such (θ, τ) angle pair was assigned to a bin pair, and the number of angle pairs assigned to each bin pair was stored in a 18×36 count matrix. Finally, the normalized count matrix was used to assign a probability to any given (θ, τ) angle pair.

List of abbreviations

- CCD: Cyclic Coordinate Descent
- DOF: Degrees Of Freedom
- FCCD: Full Cyclic Coordinate Descent
- MCMC: Markov Chain Monte Carlo
- RMSD: Root Mean Square Deviation

Authors' contributions

TH conceived the FCCD algorithm. WB implemented FCCD in the C language, and introduced various refinements and optimizations. Both authors read and approved the article.

Additional material

Additional File 1

The file FCCD.py contains an implementation of the FCCD algorithm. The program was implemented in the interpreted, object oriented language Python <http://www.python.org>. The Numeric Python package <http://numeric.scipy.org/>, a Python module that implements many advanced mathematical operations efficiently in C and FORTRAN, provided implementations of singular value decomposition and various matrix operations. In addition, the Biopython toolkit, a set of Bioinformatics modules implemented in Python, was used to represent atomic coordinates as vector objects [25]. The core of the FCCD implementation comprises only 50 lines of Python code. Numeric Python and Biopython (version 1.4b) are needed to execute the sample code.

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[<http://www.biomedcentral.com/content/supplementary/1471-2105-6-159-S1.py>]

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